LIBERTY PAPER SET

STD. 10 : Mathematics (Standard) [N-012(E)]

Full Solution

Time : 3 Hours **ASSIGNTMENT PAPER 6**

Section-A

1. (D) positive integer **2.** (C) –1 **3.** (D) 6 **4.** (B) 2500 **5.** (A) 50° **6.** (D) 2 **7.** $p = 2$ and $q = 5$ **8.** $\alpha \cdot \beta = \frac{c}{a}$ **9.** $\frac{-b}{2a}$ **10.** 1 : 1 **11.** 15 **12.** $u_i = \frac{x_i - a}{h}$ **13.** False **14.** False **15.** False **16.** True **17.** True **18.** 4 **19.** 1 **20.** 9 cm **21.** $\frac{12}{5}$ **22.** 6° **23.** (c) $\pi r^2 h$ **24.** (a) πd

Section-B

25. $w = 91 \times 5 = 455$

$y = w \times 3 = 455 \times 3 = 1365$

$z = y \times 3 = 1365 \times 3 = 4095$

$x = z \times 2 = 8190$

So, $x + y - z - w = 8190 + 1365 - 4095 - 455 = 5005$

26. $2x + 3y = 11$ ...(1)

$2x - 4y = -24$ ...(2)

As per equation (1)

$y = \frac{11 - 2x}{3}$ ...(3)

Put value of equation (3) in equation (2)

$2x - 4y = -24$

$\therefore 2x - 4\left(\frac{11 - 2x}{3}\right) = -24$

$\therefore 6x - 44 + 8x = -72$

$\therefore 6x + 8x = -72 + 44$

$\therefore 14x = -28$

$\therefore x = -2$

Put $x = -2$ in equation (3)

$y = \frac{11 - 2x}{3}$

$\therefore y = \frac{11 - 2(-2)}{3}$

$\therefore y = \frac{11 + 4}{3}$

$\therefore y = 5$

The solution : $x = -2$, $y = 5$

Now, $y = mx + 3$

$\therefore 5 = m(-2) + 3$

$\therefore 5 - 3 = -2m$

$\therefore -2m = 2$

$\therefore m = -1$

27. Let one of the odd positive integer be x

then the other odd positive integer is $x + 2$

their sum of squares = $x^2 + (x + 2)^2$

$= x^2 + x^2 + 4x + 4$

$= 2x^2 + 4x + 4$

Given that their sum of squares = 290

$2x^2 + 4x + 4 = 290$

$2x^2 + 4x = 290 - 4 = 286$

$2x^2 + 4x - 286 = 0$

$2(x^2 + 2x - 143) = 0$

$x^2 + 2x - 143 = 0$

$x^2 + 13x - 11x - 143 = 0$

$x(x + 13) - 11\ (x + 13) = 0$

$(x - 11) = 0, (x + 13) = 0$

Therefore, $x = 11$ or $- 13$

We always take positive value of x

So, $x = 11$ and $(x + 2) = 11 + 2 = 13$

Therefore, the odd positive integers are 11 and 13.

28. $2x^2 + kx + 3 = 0$

$\therefore\ a = 2,\ b = k,\ c = 3$

We know that, roots of given equation are equal.

$\therefore\ b^2 - 4ac = 0$

$\therefore\ k^2 - 4(2)(3) = 0$

$\therefore\ k^2 - 24 = 0$

$\therefore\ k^2 = 24$

$\therefore\ k = \pm 2\sqrt{6}$

29. Three-digit numbers that is divisible by 7 are : 105, 112, 119, ..., 994. which gives the AP.

$\therefore\ a = 105,\ d = 112 - 105 = 7,\ a_n = l = 994$

$a_n = a + (n - 1)d$

$\therefore\ 994 = 105 + (n - 1)7$

$\therefore\ 142 = 15 + n - 1$

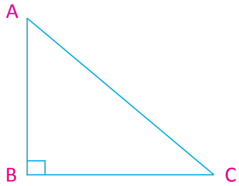
$\therefore\ 142 = 14 + n$

$\therefore\ n = 142 - 14$

$\therefore\ n = 128$

Therefore, 128 three-digit numbers are divisible by 7.

30.



In Δ ABC; $\angle B = 90°$

$3\ cot\ A = 4$

$\therefore\ cot\ A = \frac{4}{3}$

$\therefore\ \frac{AB}{BC} = \frac{4}{3}$

$\therefore\ \frac{AB}{4} = \frac{BC}{3} = k$, where k is positive real number.

$\therefore$ AB = $4k$, BC = $3k$

According to Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$

$\therefore\ AC^2 = (4k)^2 + (3k)^2$

$\therefore\ AC^2 = 16k^2 + 9k^2$

$\therefore\ AC^2 = 25k^2$

$\therefore\ AC = 5k$

$\therefore\ sin\ A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$,

$cos\ A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$,

$tan\ A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

$$LHS = \frac{1 - tan^2 A}{1 + tan^2 A} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{7}{25}$$

$$RHS = cos^2 A - sin^2 A = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$\therefore$ LHS = RHS

$$\therefore\ \frac{1 - tan^2 A}{1 + tan^2 A} = cos^2 A - sin^2 A$$

31. $sin(A - B) = \frac{1}{2}$ $\qquad cos\ (A + B) = \frac{1}{2}$

$\therefore\ sin(A - B) = sin\ 30°$ $\therefore\ cos(A + B) = cos\ 60°$

$\therefore$ A – B = 30°....(1) $\therefore$ A + B = 60° ....(2)

Adding equation (1) and (2),

$(A - B) + (A + B) = 30° + 60°$

$\therefore\ A - B + A + B = 90°$

$\therefore\ 2A = 90°$

$\therefore\ A = 45°$

Put A = 45° in equation (1),

$A - B = 30°$

$\therefore\ B = A - 30°$

$\therefore\ B = 45° - 30°$

$\therefore\ B = 15°$

Hence, A = 45° and B = 15°.

32. For such case

$\angle$ POQ + $\angle$ PTQ = 180°

$\therefore$ 110° + $\angle$ PTQ = 180°

$\therefore$ $\angle$ PTQ = 180° – 110°

= 70°

33. The edge of the cube = l = 5 cm

For hemisphere, Radius = $r = \frac{d}{2} = \frac{4.2}{2} = 2.1$ cm

The surface area of the block

= TSA of cube – base area of hemisphere + CSA of hemisphere

$= 6l^2 - \pi r^2 + 2\pi r^2$

$= 6l^2 + \pi r^2$

$= 6(5)^2 + \left(\frac{22}{7} \times 2.1 \times 2.1\right)$

= 150 + 13.86

= 163.86 cm^2

Thus, the total surface area of the block will be 163.86 cm^2.

34.

Number of mangoes (class)	Number of boxes (f_i)	x_i	u_i	$f_i u_i$
50–52	15	51	–2	–30
53–55	110	54	–1	–110
56–58	135	57 = a	0	0
59–61	115	60	1	115
62–64	25	63	2	50
Total	$\Sigma f_i = 400$	–	–	$25 = \Sigma f_i u_i$

Mean $\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$\therefore \bar{x} = 57 + \frac{25}{400} \times 3$

$\therefore \bar{x} = 57 + 0.19$

$\bar{x} = 57.19$

So, Mean number of mangoes kept in a packing box is 57.19.

Here, the step deviation method is used to find the mean.

35. We have, $\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

$\therefore$ from given values,

$\bar{x} = 30 + \frac{(-26)}{13}$

$= 30 - \frac{26}{13}$

$= 30 - 2$

$\bar{x} = 28$

36. Total number of balls in the bag = 3 + 5 = 8

$\therefore$ The total number of autcomes = 8

(i) Suppose event A drawn ball is red.

$\therefore$ P(A) = $\frac{\text{Number of red ball}}{\text{Total number of ball}}$

$\therefore$ P(A) = $\frac{3}{8}$

(ii) Suppose event B drawn ball is not red.

$\therefore$ P(B) = $\frac{\text{Then number of balls that are not red}}{\text{Total number of ball}}$

$\therefore$ P(B) = $\frac{8-3}{8}$

$\therefore$ P(B) = $\frac{5}{8}$

37. Possible outcomes in throwing a die = 6 (1, 2, 3, 4, 5, 6)

(i) Suppose A be the event getting a number multiple of 3 on die.

These are 2 numbers 3 and 6 which multiple of 3 among 1 to 6

The number of outcomes favourable to A = 2

$\therefore$ P(A) = $\frac{2}{6} = \frac{1}{3}$

(ii) Suppose B be the event getting the number 2 on die.

The number 2 comes only one time.

The number of outcomes favourable to B = 1

$\therefore$ P(B) = $\frac{1}{6}$

Section-C

38. α, β roots of $f(x) = kx^2 + 4x + 4$

Given $\alpha^2 + \beta^2 = 24$

We know $\alpha + \beta = \frac{-b}{a} = \frac{-4}{k}$

$\alpha\beta = \frac{c}{a} = \frac{4}{k}$

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\left(\frac{-4}{k}\right)^2 = 24 + 2\left(\frac{4}{k}\right)$

$\frac{4^2}{k^2} = 24 + 2\left(\frac{4}{k}\right)$

$16 = 24k^2 + 8k$

$2 = 3k^2 + k$

$0 = 3k^2 + k - 2$

$0 = 3k^2 + 3k - 2k - 2$

$0 = 3k(k + 1) - 2(k + 1)$

$0 = (k + 1)(3k - 2)$

$\therefore$ $k = -1, \frac{2}{3}$

39. Let the quadratic polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$\therefore\ \alpha + \beta = \frac{1}{4}$ and $\alpha\beta = -\frac{1}{4}$

$\therefore\ \frac{-b}{a} = \frac{1}{4}$ and $\frac{c}{a} = -\frac{1}{4}$

$\therefore$ if $a = 4,\ b = -1$ and $c = -1$

So, one quadratic polynomial which fits the given conditions is $4x^2 - x - 1$. You can check that any other quadratic polynomial that fits these conditions will be of the form $k(4x^2 - x - 1)$, where k is real.

40. Given AP is 3, 8, 13, ..., 253

$\therefore\ a = 3,\ d = 8 - 3 = 5,\ a_n = 253$

$a_n = a + (n - 1)d$

$\therefore\ 253 = 3 + (n - 1)5$

$\therefore\ 253 - 3 = (n - 1)5$

$\therefore\ \frac{250}{5} = n - 1$

$\therefore\ n - 1 = 50$

$\therefore\ n = 51$

Therefore, $n = 51$, the last 20th term of the given series is the 32nd term.

$a_{32} = a + 31d$

$= 3 + 31(5)$

$= 3 + 155$

$a_{32} = 158$

Therefore, the 20th term from the last term of the AP is 158.

41. $a_{12} = 37,\ d = 3,\ a =$ ______, $S_{12} =$ ______

Now, $a_{12} = 37$

$\therefore\ a + 11d = 37$

$\therefore\ a + 11(3) = 37$

$\therefore\ a + 33 = 37$

$\therefore\ a = 37 - 33$

$\therefore\ a = 4$

$S_n = \frac{n}{2}\,[2a + (n - 1)d]$

$\therefore\ S_{12} = \frac{12}{2}\,[2(4) + (12 - 1)(3)]$

$= 6\,[8 + 33]$

$= 6 \times 41$

$\therefore\ S_{12} = 246$

42. $AP = \frac{3}{7}\,AB$

$\therefore\ \frac{AB}{AP} = \frac{7}{3}$

$\therefore\ \frac{AB - AP}{AP} = \frac{7 - 3}{3}$

$\therefore\ \frac{PB}{AP} = \frac{4}{3}$ $(\because\ A - P - B)$

$\therefore\ \frac{AP}{PB} = \frac{3}{4}$

$\therefore\ AP : PB = m_1 : m_2 = 3 : 4$

Thus, point P (x, y) divides the line segment connecting A (–2, –2) and B (2, –4) in the ratio $m_1 : m_2 = 3 : 4$.

∴ The co-ordinates of the dividing point P

$$= \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$\therefore (x, y) = \left(\frac{3(2)+4(-2)}{3+4}, \frac{3(-4)+4(-2)}{3+4}\right)$$

$$\therefore (x, y) = \left(\frac{6-8}{7}, \frac{-12-8}{7}\right)$$

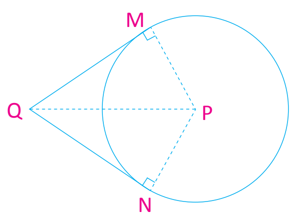
$$\therefore (x, y) = \left(-\frac{2}{7}, -\frac{20}{7}\right)$$

Hence, the co-ordinates of P $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

43. Given : O (p, r) has QM and QN tangents drawn to a circle from an external point Q.

To prove : QM = QN

Figure :



Proof : Join PQ, PM and PN. Then ∠PMQ and ∠PNQ are right angles because these angles are between the radii and tangents and according to theorem 10.1 they are right angles.

Now, in ΔPMQ and ΔPNQ,

PM = PN (Radii of the same circle)

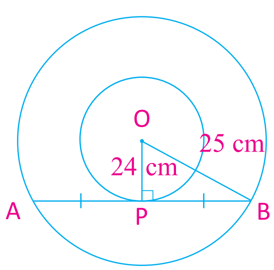
PQ = PQ (Common)

∠PMQ = ∠PNQ (Right angle)

Therefore, Δ PMQ ≅ Δ PNQ (RHS)

∴ QM = QN (CPCT)

44.



In two P concentric circles a chord of a circle with large radius touches a small circle.

Large circle radii PB = 25 cm

Small circle radii PM = 24 cm

PM ⊥ AB, ∠PMB = 90

∴ According to any pythagoras Theorem,

$PM^2 + MB^2 + PB^2$

∴ $(24)^2 + MB^2 = (25)^2$

∴ $576 + MB^2 = 625$

∴ $MB^2 = 625 - 576$

∴ $MB^2 = 49$

∴ MB = 7 cm

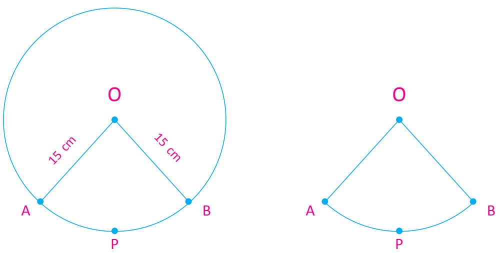
M is a midpoint of chord AB

AB = 2 MB

= 2 × 7

AB = 14 cm

45. Here, $r = 15, \quad \theta = 60°$



Area of smaller portion $= \dfrac{\pi r^2 \theta}{360}$

$= \dfrac{3.14 \times 15 \times 15 \times 60}{360}$

$= \dfrac{3.14 \times 15 \times 15 \times 60}{100 \times 60 \times 6}$

$= \dfrac{314 \times 5 \times 3 \times 15}{100 \times 6}$

$= \dfrac{157 \times 2 \times 5 \times 3 \times 15}{100 \times 2 \times 3}$

$= \dfrac{157 \times 5 \times 15}{100}$

$= \dfrac{11775}{100}$

$\therefore$ Area of smaller portion = 117.75 cm^2

Area of larger portion

= Area of full circle – Area of smaller portion

$= \pi r^2 - 117.75$

$= (3.14 \times 15 \times 15) - 117.75$

$= \left(\dfrac{314 \times 5 \times 3 \times 15}{100}\right) - 117.75$

$= \left(\dfrac{157 \times 2 \times 5 \times 3 \times 15}{10 \times 10}\right) - 117.75$

$= \dfrac{157 \times 3 \times 15}{10} - 117.75$

$= 706.50 - 117.75$

= 588.75 cm^2

$\therefore$ Area of larger portion is 588.75 cm^2.

46. Here, total number of cards = 52

(i) Suppose event A is the king of red colour (king of red and king of diamond).

$\therefore P(A) = \dfrac{\text{Number of kings of red colour}}{\text{Total number of outcomes}}$

$\therefore P(A) = \dfrac{2}{52}$

$\therefore P(A) = \dfrac{1}{26}$

(ii) Suppose event B is a red face card.

$\therefore P(B) = \dfrac{\text{Number of red face card}}{\text{Total number of outcomes}}$

$\therefore P(B) = \dfrac{6}{52}$

$\therefore P(B) = \dfrac{3}{26}$

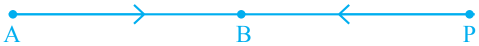
(iii) Suppose event C is a spade card (13).

$\therefore P(C) = \dfrac{\text{Number of getting a spade card}}{\text{Total number of outcomes}}$

$\therefore P(C) = \dfrac{13}{52} = \dfrac{1}{4}$

Section-D

47. Let the speed of car A is x km/h and car B is y km/h $(x > y)$



If the cars move in the same directions, they meet each other in 5 hours at point P.

Distance = Time x speed

In 5 hours car A running distance AP = $5x$ km and in 5 hours car B running distance BP = $5y$ km.

$\therefore$ Now, AB = 100 km

$\therefore$ AP – BP = 100 (A – B – P)

$\therefore$ $5x - 5y = 100$

$\therefore$ $x - y = 20$....(1)

If the cars move towards each other in 60 minutes = 1 hours at point Q.

In 1 hours car A running distance AQ = x km and in 1 hours car B running distance BQ = y km.

$\therefore$ Now, AB = 100 km

$\therefore$ AQ + BQ = 100 (A – Q – B)

$\therefore$ $x + y = 100$ ....(1)

Add equation 1 & 2

$x - y = 20$

$x + y = 100$

$2x = 120$

$\therefore$ $x = \frac{120}{2}$

$\therefore$ $x = 60$ km/h

From (2)

$\therefore$ $60 + y = 100$

$\therefore$ $y = 100 - 60$

$\therefore$ $y = 40$ km/h

$\therefore$ The speed car A is 60 km/h and car B is 40 km/h.

48. Suppose, present age of Jacob is x year and present age of his son is y year.

After five years,

Age of Jacob is $(x + 5)$ years

Age of his son is $(y + 5)$ years.

According to the first condition,

$x + 5 = 3(y + 5)$

$\therefore$ $x + 5 = 3y + 15$

$\therefore$ $x - 3y = 10$ ...(1)

$\therefore$ $x = 3y + 10$ ...(2)

Before 5 years,

Age of Jacob is $(x - 5)$ years

Age of his son is $(y - 5)$ years.

According to the second condition,

$x - 5 = 7(y - 5)$

$\therefore$ $x - 5 = 7y - 35$

$\therefore$ $x - 7y = -30$ ...(3)

Put value of equation (2) in equation (3)

$x - 7y = -30$

$\therefore\ 3y + 10 - 7y = -30$

$\therefore\ 3y - 7y = -30 - 10$

$\therefore\ -4y = -40$

$\therefore\ y = 10$

Put $y = 10$ in equation (2)

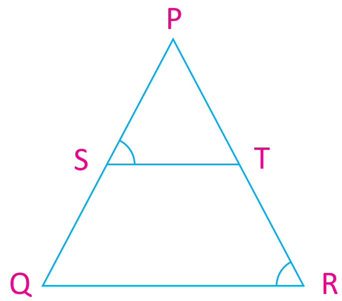
$x = 3y + 10$

$\therefore\ x = 3(10) + 10 = 30 + 10 = 40$

$\therefore\ x = 40$

Hence, the present age at Jacob's and his son is 40 years and 10 year.

49.



It is given that $\frac{PS}{SQ} = \frac{PT}{TR}$

$\therefore$ ST || QR (Theorem 6.1)

$\therefore\ \angle PST = \Delta PQR$ (corresponding angle) ...(1)

Also it is given that, $\angle PST = \angle PRQ$ ...(2)

As per eq^n. (1) & (2),

$\angle PRQ = \angle PQR$

$\therefore$ PQ = PR (sides opposite the equal angles)

i.e., ΔPQR is an isosceles triangle.

50. In the proof below, the point D is on the side BC of ΔABC such that $\angle ADC \cong \angle BAC$, than prove that $CA^2 = CB \cdot CD$.

Given: point D is on the side BC of ΔABC such that $\angle ADC \cong \angle BAC$.

To Prove : $\underline{CA^2 = CB \cdot CB}$.

Proof : ΔCDA and ΔCAB have $\angle ADC = \angle \underline{BAC}$.

(Given)

And $\angle ACD = \angle BCA$ [Same angle (common)]

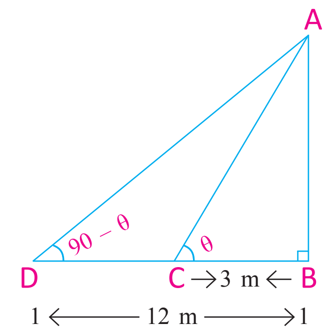
$\therefore$ AA condition by condition, $\Delta CDA \sim \Delta CAB$

$\therefore\ \frac{CD}{CA} = \frac{CA}{CB}$

$\therefore\ CB \cdot CD = CA \cdot CA$

$\therefore\ CA^2 = CB \cdot CD$

51. Here AB is the tower. C and D are two observation point respectively 3m and 12m from bottom of tower.



In ΔABC $\angle B = 90°$

$\therefore$ BC = 3 m and BD = 12 m

$\therefore$ Suppose $\angle ACB = \theta$, then $\angle ADB = 90 - \theta$ (Complementry angles)

In right atriangle ABC

$tan\theta = \dfrac{AB}{BC}$

$\therefore tan\theta = \dfrac{AB}{3}$ ....(1)

In right atriangle ABD

$tan\ (90 - \theta) = \dfrac{AB}{BD}$

$\therefore cot\theta = \dfrac{AB}{12}$ ....(2) $(\therefore tan\ (90 - \theta) = cot\ \theta)$

From (1) & (2)

$tan\ \theta \bullet cot\ \theta = \dfrac{AB}{3} \times \dfrac{AB}{12}$

$\therefore 1 = \dfrac{AB^2}{36}$ $(\therefore tan\ \theta \bullet cot\ \theta = 1)$

$\therefore 36 = AB^2$

$\therefore AB = 6$ m

$\therefore$ The height of the tower = 6 m

52.

Hemisphere	Cone	Cylinder
$r = 60$ cm	$r = 60$ cm	$r = 60$ cm
	$h = 120$ cm	H = 180 cm

Total volume of solid = Volume of hemisphere + Volume of cone

$= \dfrac{2}{3}\ \pi r^3 + \dfrac{1}{3}\ \pi r^2 h$

$= \dfrac{1}{3}\ \pi r^2 (2r + h)$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times 60 \times 60 \times (2 \times 60 + 120)$

$= \dfrac{22 \times 20 \times 60}{7} \times (120 + 120)$

$= \dfrac{22 \times 20 \times 60 \times 240}{7}$

$= \dfrac{6336000}{7}$ cm^3

Volume of cylinder $= \pi r^2 H$

$= \dfrac{22}{7} \times 60 \times 60 \times 180$

$= \dfrac{14256000}{7}$ cm^3

The volume of water left in the cylinder

= Volume of cylinder – Volume of solid

$= \dfrac{14256000}{7} - \dfrac{6336000}{7}$

$= \dfrac{14256000 - 6336000}{7}$

$= \dfrac{7920000}{7}$

= 1131428.57 cm^3

$= \dfrac{1131428.57}{1000000}$ m^3

= 1.131 m^3 (Approx)

53. Cylinder — Hemisphere

$h = 1.45$ m $= 145$ cm — $r = 30$ cm

$r = 30$ cm

The total surface area of the bird-bath

= CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r^2$

$= 2\pi r\,(h + r)$

$= 2 \times \frac{22}{7} \times 30 \times (145 + 30)$

$= 2 \times \frac{22}{7} \times 30 \times 175$

$= 33000 \text{ cm}^2$

Thus, TSA of the bird-bath is $33000 \text{ cm}^2 = 3.3 \text{ m}^2$

54.

class	frequency (f_i)	cumulative frequency cf
10 – 20	12	12
20 – 30	30	$12 + 30 = 42$
30 – 40	a	$42 + a$
40 – 50	65	$42 + a + 65 = a + 107$
50 – 60	b	$a + b + 107$
60 – 70	25	$a + b + 107 + 25 = a + b + 132$
70 – 80	18	$a + b + 132 + 18 = a + b + 150$

Here, $\sum fi = 230$

but, $\sum fi = a + b + 150$

$\therefore\ a + b + 150 = 230$

$\therefore\ a + b = 230 - 150$

$\therefore\ a + b = 80$ ....(1)

The median is 46, which is lies in the class 40 – 50.

So, median class is 40 – 50.

$l = 40,\ f = 65,\ cf = a + 42,\ h = 10,\ \frac{n}{2} = \frac{230}{2} = 115$

Median $M = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$\therefore\ 46 = 40 + \left(\frac{115 - (a + 42)}{65}\right) \times 10$

$\therefore\ 46 - 40 = \left(\frac{115 - a - 42}{13}\right) \times 2$

$\therefore\ 6 = \left(\frac{73 - a}{13}\right) \times 2$

$\therefore\ \frac{\overset{3}{\cancel{6}} \times 13}{2} = 73 - a$

$\therefore\ 39 = 73 - a$

$\therefore\ a = 73 - 39$

$\therefore\ a = 34$

From (1)

$34 + b = 80$

$\therefore\ b = 80 - 34$

$\therefore\ b = 46$

Hence $a = 34$ and $b = 46$